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Stimulated emission processes near a black hole

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Abstract. The amplification of the unstable photon orbits in the Schwarzschild metric near $r = 3M$ by stimulated emission is discussed. The behaviour of these orbits is studied in detail, and it is shown that orbits which circle the black hole a large number of times can be greatly amplified by stimulated emission. The effects of the Doppler and gravitational red shifts are calculated. A mechanism by which a population inversion can be created in infalling atoms is suggested. An upper bound on the energy released by this process is discussed, and it is shown that the energy output by stimulated emission is a very small fraction of the total energy released by accretion of matter onto a black hole.

1. Introduction

Electromagnetic processes near black holes have been extensively studied in recent years both from the viewpoint of fundamental physics and that of astrophysics. It has been suggested that stimulated emission processes could occur in the strong gravitational field of a black hole, and that such processes could explain the unusual features of the object SS433 (Zhi and Ruffini 1979, Ruffini and Stella 1980). In this paper, a particular example of a stimulated emission process near a Schwarzschild black hole will be considered.

It is well known that there exist unstable circular photon orbits in the Schwarzschild metric at $r = 3M$. A photon in such an orbit which is subjected to a small perturbation will either spiral out to infinity or down across the event horizon. A photon which eventually reaches infinity can, however, orbit the black hole many times near $r = 3M$. This suggests the possibility of producing laser action by amplifying these photon modes. If an active material (i.e. one with suitable population inversions) were present at $r = 3M$, it would be possible to amplify these modes and produce emission of coherent radiation by the black hole. This type of stimulated emission by atoms in the vincinity of a black hole is not to be confused with the classical amplification of waves by a rotating black hole (Press and Teukolsky 1972) or the quantum creation of particles by small black holes (Hawking 1975).

In § 2 some of the properties of photon orbits near $r = 3M$ are considered. In § 3 the effects of the Doppler and gravitational red shifts upon the frequency detected by an observer at infinity are calculated. Some aspects of the hydrodynamics of matter accreting onto a black hole are considered in § 4, and a possible mechanism for creation of a population inversion is proposed. Limitations upon the energy output by stimulated emission are discussed in § 5.

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2. Photon orbits near $r = 3M$

In this section, we consider some of the properties of null geodesics in the vicinity of $r = 3M$. The null geodesics in the Schwarzschild spacetime of mass M are solutions of (Misner *et al* 1973)

$$(\mathrm{d}u/\mathrm{d}\phi)^2 + u^2(1-2u) = (M/b)^2 \tag{1}$$

where $u = M/r$, ϕ is an azimuthal angular coordinate and b is the impact parameter. The unstable circular orbits at $r = 3M$ are solutions of (1) for $b = \sqrt{27}M$. We wish to consider nearby orbits. Let

$$b = (\sqrt{27} + \varepsilon)M. \tag{2}$$

From (1) we see that the turning point of an orbit occurs at $u = u_0$, where

$$2u_0^3 - u_0^2 + (M/b)^2 = 0. \tag{3}$$

Let $u_0 = \frac{1}{3} - \delta$. If we expand (3) to lowest order in ε and δ, we find that

$$\delta = 2^{1/2} 27^{-3/4} \varepsilon^{1/2}. \tag{4}$$

Let $u = u_0 - \xi$; then (1) becomes for small ξ

$$\mathrm{d}\xi/\mathrm{d}\phi = (2u_0 - 6u_0^2)^{1/2}\xi^{1/2} \sim (2\delta)^{1/2}\xi^{1/2}$$

which has the solution

$$\phi = (2/\delta)^{1/2}\xi^{1/2} = 2^{1/4}27^{3/8}\varepsilon^{-1/4}\xi^{1/2} \sim 4.1\varepsilon^{-1/4}\xi^{1/2}. \tag{5}$$

This result describes the motion of a photon near the turning point of its orbit.

For small $\varepsilon > 0$, the photon will orbit many times near $r = 3M$ before moving off to infinity. The number of revolutions which the photon makes between the turning point and $r = M(u_0 - \xi)^{-1} \sim Mu_0^{-1}(1 + \xi u_0^{-1})$ is

$$n = \phi/2\pi \sim 0.65\varepsilon^{-1/4}\xi^{1/2}. \tag{6}$$

For the sake of illustration, let $\xi = 10^{-4}$. Then we have $n \sim 6.5 \times 10^{-2}\varepsilon^{-1/4}$. Hence for $\varepsilon = 10^{-8}$, $n \sim 6.5$, for $\varepsilon = 10^{-12}$, $n \sim 65$, etc. The probability that a photon emitted randomly by an atom near the black hole or shot at the black hole from infinity will find itself in one of these orbits is proportional to ε. Thus a tightly spiralling orbit would normally be very difficult to attain. However, if photon modes are being amplified by the presence of an active material, those modes associated with multiple revolutions will be favoured. For example, note that $(1.17)^{65-6.5} = 10^4$. If the net gain per revolution is greater than 17%, modes with $\varepsilon = 10^{-12}$ in the above example will be amplified by a factor of more than 10^4 compared with $\varepsilon = 10^{-8}$ modes. Consequently, the former modes will more than overcome the initial handicap arising from their small value of ε.

The value of ξ which will be important for our purposes is determined by the line width of the atomic transition being employed. As the photons spiral away from $r = 3M$, Doppler and gravitational red shifts will eventually cause them to become out of tune with the transition and amplification will cease. This will be discussed in more detail in the next section. If ρ is the net amplification per revolution, then the total amplification possible is

$$R = \rho^{2n} \sim \rho^{1.3\varepsilon^{-1/4}\xi^{1/2}} \tag{7}$$

where n is given by (6).

The factor of two which has appeared in the exponent comes from the fact that there are $2n$ revolutions between $u = u_0$ and $u = u_0 - \xi$ counting both approach to and recession from the turning point. Equation (7) tells us that for any $\rho > 1$, one can obtain very large total amplification in those modes for which ε is sufficiently small. This means that stimulated emission into these modes will be a very efficient mechanism for transferring energy from the active material to the electromagnetic field. The output of the laser will be limited primarily by the size of the population inversion.

3. Doppler and gravitational red shifts

In this section we wish to consider the combined effects of the Doppler and gravitational red shift which determine the frequency at which the radiation is detected by an observer at infinity. Let k^μ be the photon wavevector, u^μ be the four-velocity of the observer, and v^μ be the four-velocity of the emitter. Then the frequency of emission, measured in the rest frame of the emitting atom, is

$$\omega_e = (k_\mu v^\mu)_r \tag{8}$$

where r is the radial coordinate at which emission occurs. The frequency observed at infinity is

$$\omega_{ob} = (k_\mu u^\mu)_\infty. \tag{9}$$

Geodesics for massive particles in the Schwarzschild metric

$$ds^2 = (1-2M/r)\,dt^2 - (1-2M/r)^{-1}\,dr^2 - r^2(d\theta^2 + \sin^2\theta\,d\phi^2) \tag{10}$$

are given by (Misner *et al* 1973)

$$\begin{aligned} v^r &= dr/d\tau = \pm[\tilde{E}^2 - (1-2M/r)(1+\tilde{L}^2/r^2)]^{1/2} \\ v^\phi &= d\phi/d\tau = \tilde{L}/r^2 \\ v^t &= dt/d\tau = \tilde{E}(1-2M/r)^{-1} \end{aligned} \tag{11}$$

where $\tilde{E} = E/\mu$, $\tilde{L} = L/\mu$, E is the energy of the particle, L is its angular momentum and μ is its rest mass. The motion of a massless particle is obtained by letting $\mu \to 0$ and setting $\lambda = \tau\mu$. Then v^μ becomes k^μ, where

$$\begin{aligned} k^r &= dr/d\lambda = \pm[E^2 - (1-2M/r)L^2/r^2]^{1/2} \\ k^\phi &= d\phi/d\lambda = L/r^2 \end{aligned}$$

and

$$k^t = dt/d\lambda = E(1-2M/r)^{-1}. \tag{12}$$

Let us first consider the emission of photons by radially infalling atoms. Then we set $\tilde{L} = 0$ in (11) and take the minus sign for v^r. If the atoms fall from infinity, then $\tilde{E} > 1$, whereas if they fall from rest at a finite radius, then $\tilde{E} < 1$. Because the photons are emitted to infinity, we take the plus sign in k^r. The impact parameter is $b = L/E$, and for the present purposes may be approximated by $\sqrt{27}M$, which enables us to eliminate L in (12). We find that

$$k^\mu v_\mu = E(1-2M/r)^{-1}\{\tilde{E} + [1-(27M^2/r)(1-2M/r)]^{1/2}[E^2 - (1-2M/r)]^{1/2}\}. \tag{13}$$

If the observer at infinity is at rest with respect to the black hole, then $u^\mu = (1, 0, 0, 0)$ and $\omega_{ob} = E$. Setting $r = 3M$ in (13) then yields

$$\omega_{ob}/\omega_e = (3\tilde{E})^{-1}. \tag{14}$$

Another possible situation is that in which the atoms fall out of the marginally stable orbit at $r = 6M$. This would be the case for infall from an accretion disc, which would be expected to have its inner edge at $r = 6M$. This orbit is characterised by $\tilde{L} = 2\sqrt{3}M$ and $\tilde{E} = \sqrt{8}/3$. We take the orbit of the atoms to lie in the equatorial plane ($\theta = \frac{1}{2}\pi$) and the orbit of the photons to be inclined at an angle of ψ with respect to the equatorial plane. Then

$$k^\theta = -(L/r^2)\sin\psi \qquad k^\phi = (L/r^2)\cos\psi \tag{15}$$

and k^r and k^t are as given in (12). With the above values of $\tilde{L}$ and $\tilde{E}$, and with $b = L/E = \sqrt{27}M$, we obtain

$$\begin{aligned} k^\mu v_\mu = E(1-2M/r)^{-1}\{&2\sqrt{2}/3 + [1-(27M^2/r^2)(1-2M/r)]^{1/2} \\ &\times[8/9-(1-2M/r^2)(1+12M^2/r^2)]^{1/2} - (6\sqrt{6}M^2/r^2)(1-2M/r)\cos\psi\} \end{aligned} \tag{16}$$

and setting $r = 3M$ we find

$$\frac{\omega_{ob}}{\omega_e} = \frac{\sqrt{3}}{2\sqrt{2}(\sqrt{3}-\cos\psi)}. \tag{17}$$

Note that ψ is here defined such that $\psi = 0$ corresponds to emission in the forward (co-rotating) direction and $\psi = \pi$ corresponds to emission in the backward (counter-rotating) direction.

We may also compute the value of ξ which is to be used in (6). Recall that it is determined by the distance Δr by which the photons may move away from $r = 3M$ and still remain tuned to the laser transition. Let us consider orbits for which the turning point is very close to $r = 3M$ (i.e. small ε), so that we may take $u_0 = \frac{1}{3}$, and

$$\xi = u_0^2\Delta r = \tfrac{1}{9}\Delta r. \tag{18}$$

The frequency of photons in the rest frame of the emitting atoms is $k^\mu v_\mu$, so if $\Delta\omega$ is the width of the laser transition, Δr is given by

$$\Delta r = \Delta\omega(\mathrm{d}(k^\mu v_\mu)/\mathrm{d}r)^{-1}_{r=3M}. \tag{19}$$

Thus from (13) or (16), we may compute ξ.

4. Population inversion

An essential requirement for laser action is the presence of a population inversion, which can only arise through a departure from thermal equilibrium. In this section we wish to note that such departures from equilibrium can arise naturally as a consequence of the infall of matter onto a black hole. Consider matter which falls out of the marginally stable orbit at $r = 6M$ and streams along a geodesic across the event horizon. One expects a fluid element to undergo either expansion or contraction during infall. To study this, let us compute the trace of the expansion tensor (Hawking and Ellis 1973)

$$\Theta = h^{\alpha\beta}v_{\alpha;\beta} \tag{20}$$

where v^α is the four-velocity of the infalling matter and $h^{\alpha\beta}$ is the projection operator onto the subspace orthogonal to v and is given by

$$h^{\alpha\beta} = g^{\alpha\beta} - v^\alpha v^\beta. \tag{21}$$

Here v^α is given by (11) with $\tilde{L} = 2\sqrt{3}M$, $\tilde{E} = \sqrt{8}/3$, and v^r negative. After some calculation, one finds that for these geodesics

$$\Theta = \frac{M}{r}\left(1 - \frac{12M}{r^2} + \frac{36M^2}{r}\right)\left[\frac{8}{9} - \left(1 - \frac{2M}{r}\right)\left(1 + \frac{12M^2}{r^2}\right)\right]^{-1/2}$$
$$-\frac{2}{r}\left[\frac{8}{9} - \left(1 - \frac{2M}{r}\right)\left(1 + \frac{12M^2}{r^2}\right)\right]^{1/2}. \tag{22}$$

For $r < 6M$, $\Theta > 0$. Because

$$\Theta = V^{-1}\,\mathrm{d}V/\mathrm{d}\tau \tag{23}$$

where V is a fluid volume element and τ is the proper time along the geodesic, positive Θ corresponds to expansion of the fluid during infall. At first sight one might expect compression rather than expansion. However, two particles which begin falling inward from the same point in space at slightly different times will move apart from one another as a result of the tidal forces. It is this effect which produces the net expansion along these particular geodesics. Note that we have specified a fluid element which is constrained to the equatorial plane ($\theta = 0$). Thus the geodesics of the congruence involved in (20) lie in or infinitesimally close to this plane and consequently do not intersect.

The expansion Θ increases from $\Theta = 0$ at $r = 6M$ to $\Theta \simeq 0.11\ M^{-1}$ at $r = 3M$ during a proper time of the order of M. This expansion will cause a departure from thermal equilibrium which is capable of producing population inversions. If one has a transition in which the upper level is metastable and decays by a radiative transition whereas the lower level is strongly coupled to other states of the system, then after a rapid expansion and hence a drop in temperature, the lower level rapidly relaxes to its new equilibrium population but the upper level tends to remain at its old equilibrium population. This can result in a population inversion, and is the pumping method used in gas dynamic lasers such as the CO_2 laser.

5. Discussion

In the previous sections, we have considered the possibility of stimulated emission near a black hole and have argued that the creation of a population inversion and the amplification of modes associated with orbits near $r = 3M$ are theoretical possibilities. If the resulting radiation could be detected, it would constitute very strong evidence for the presence of a black hole. No other known object is capable of producing circular photon orbits or the large gravitational red shifts discussed in § 3.

It is, however, unlikely that the energy output from the process discussed in this paper is sufficiently large to be directly detectable. It is well known that the binding energy of the marginally stable orbit at $r = 6M$ is about 0.06 of the rest mass energy of the particle. Hence the total luminosity of an accreting Schwarzschild black hole should be of the order of 6% of the mass accretion rate. However the maximum possible energy output in stimulated emission corresponds to one photon per accreted atom.

Thus the ratio of the stimulated emission luminosity to the total luminosity is

$$\frac{L_{\text{laser}}}{L_{\text{total}}} < \frac{\omega}{0.06\, mc^2} \tag{24}$$

where ω is the red-shifted frequency of the laser transition and m is the mass of the atom. With $\omega \simeq 1$ eV and $mc^2 = 10^9$ eV, $L_{\text{laser}}/L_{\text{total}} < 10^{-8}$. Other considerations, such as the fact that not all atoms will participate in the stimulated emission, will reduce this ratio further.

Consequently this process is not likely to be of direct astrophysical interest. It is, however, indicative of the types of exotic physical processes which are possible in the vicinity of a black hole.

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